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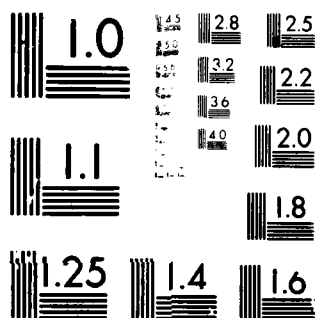
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Master's

THESIS

6 AN INVESTIGATION OF  
MAGNETO-ACOUSTIC EFFECTS IN CONDUCTIVE FLUIDS.

by

10 Robert Franz/Klaus

11 Dec 79

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Thesis Advisor:

P.H. Moose

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#20 - ABSTRACT - CONTINUED

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An Investigation of  
Magneto-Acoustic Effects in Conductive Fluids

by

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Lieutenant, United States Navy  
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Submitted in partial fulfillment of the  
requirements for the degree of

MASTER OF SCIENCE IN ENGINEERING ACOUSTICS

from the  
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### ABSTRACT

An acoustic wave propagating in a conductive medium in the presence of a magnetic field will induce electric potential fields and currents, and, associated with the electric currents, secondary magnetic fields. This interaction is of interest because it occurs in acoustic propagation through sea water in the presence of the earth's magnetic field, and at ultra-low frequency (ULF), acoustic waves should be detectable. A Green's function solution in an infinite medium is developed, with an infinite magnetic field and an infinite plane acoustic wave, from which it can be seen that the magneto-acoustic coupling strength varies linearly with the strength of the acoustic wave and with the magnetic field, and is inversely proportional to frequency. An experiment is described and analyzed in which this interaction is verified for a specific laboratory geometry to within an order of magnitude. Possible sources of the discrepancy in the magnitude of the effect are discussed, and some potential refinements for future experimentation are also noted.

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## I. INTRODUCTION

It has been suggested by previous investigations that an acoustic wave propagating in a conductive medium in the presence of a strong magnetic field will induce electric fields. Further, these electric fields can generate magnetic waves due to their associated currents. These, in turn, are capable of interacting with the original acoustic wave.

In 1952, Anderson<sup>1</sup> analyzed longitudinal magneto-hydrodynamic effects. Sound waves propagating in a conductive medium will undergo a change in velocity due to the constraining forces acting on the fluid from the magnetic field. His paper developed a theoretical model for predicting the dispersion and absorption of the initiating acoustic wave in media of finite conductivity.<sup>2</sup>

After a long pause in the literature, Bird<sup>3</sup> wrote a paper in 1977 which investigated hydromagnetism from submerged acoustic sources. He introduced the term "sonomagnetic

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<sup>1</sup>Neal S. Anderson, "Longitudinal Magneto-Hydrodynamic Waves," Journal of the Acoustical Society of America (JASA), May, 1953.

<sup>2</sup>Kornhauser ("Magnetic Damping of Acoustic Waves in Conducting Media," JASA, July, 1953) pointed out an unfortunate numerical error which invalidated the quantitative results of Anderson.

<sup>3</sup>J. F. Bird, "Hydromagnetism Induced by Submerged Acoustic Sources: Sonomagnetic Pseudoradiation," JASA, November, 1977.

pseudoradiation" to describe the effect; due to the characteristically weak nature of the induced magnetic fields, they do not propagate effectively of themselves, but are carried along by the propagating acoustic wave. The work develops in some detail a theoretical model for a semi-infinite conductive medium for several source geometries.

However, apparently these interactions have never been observed in a laboratory experiment. The objective of the research reported here has been to verify the existence of the magneto-acoustic effect and compare its strength with theoretical prediction.

The phenomenon is of interest for a variety of potential applications, some of which are noted here. There is a continuing interest in the attenuation of sound in seawater, especially at the lower frequencies. As will be observed later, the coupling of energy from the acoustic wave into an electromagnetic field is more pronounced at lower frequencies, and this interaction could possibly account for some acoustic attenuation.

Also of interest is the possibility that this phenomenon is a source of ultra-low frequency (ULF) noise in geomagnetic measurements at sea.

Another facet of interest is the potential use of this coupling effect as a means of observing low frequency acoustic waves by their electric or magnetic signatures. Since this effect is sensitive to the acoustic velocity field as a vectorial quantity, it provides directional information from a

point measurement. The possible benefits of observing ULF acoustic waves with an electromagnetic antenna, as opposed to large conventional acoustic arrays, are very appealing.

Before any of these postulated applications can be attempted, however, there is a need to experimentally verify the basic phenomenon of magneto-acoustic interactions. The objective of this thesis is to provide a preliminary verification of the existence of the coupling effect. To accomplish this, a general solution in an infinite medium is first examined, and then a laboratory experiment exploring the phenomenon is reported and discussed.

## II. GENERAL THEORY

This section reviews and discusses a solution for an acoustic plane wave in an infinite medium, from which several significant characteristics of the interaction are evident.

First, consider a magnetic field of the form

$$\bar{B} = \bar{B}_0 e^{i\omega t} \quad (1)$$

so that

$$\frac{\partial \bar{B}}{\partial t} = i \omega \bar{B} \quad (2)$$

Then, from Maxwell's equations, for a linear medium,

$$\nabla \times \bar{E} = -i \omega \bar{B} \quad (3)$$

and

$$\nabla \times \frac{\bar{B}}{\mu} = \bar{J} + \frac{\partial \bar{D}}{\partial t} \quad (4)$$

$\bar{J}$  is the normal current density,  $\bar{D}$  is the displacement current, and  $\mu$  is the permittivity. However, for frequencies less than about  $10^9$  Hz, the displacement current is negligible, so that

$$\nabla \times \frac{\bar{\mathbf{B}}}{\mu} = \bar{\mathbf{J}} = \bar{\mathbf{J}}_0 + \bar{\mathbf{J}}_1 = \bar{\mathbf{J}}_0 + \sigma \bar{\mathbf{E}}, \quad (5)$$

in the form of a source current and an induced current ( $\sigma$  is the conductivity of the solution). This can also be written

$$\nabla \times \bar{\mathbf{B}} = \mu \bar{\mathbf{J}}_0 + \mu \sigma \bar{\mathbf{E}}. \quad (6)$$

Taking the curl of both sides of equation (6) yields

$$-\nabla^2 \bar{\mathbf{B}} = \mu \sigma (\nabla \times \bar{\mathbf{E}}) + \mu (\nabla \times \bar{\mathbf{J}}_0); \quad (7)$$

and combining equation (7) with equation (3) above results in

$$-\nabla^2 \bar{\mathbf{B}} - \gamma^2 \bar{\mathbf{B}} = -\mu (\nabla \times \bar{\mathbf{J}}_0), \quad (8)$$

where

$$\gamma = (i\omega\mu\sigma)^{1/2}$$

is the complex propagation constant. This is a standard wave equation in  $\bar{\mathbf{B}}$ , the magnetic field strength.

Now introduce a plane acoustic wave propagating in the +z direction and assume a DC magnetic field,  $F$ , in the y-z plane, so that

$$F_y = F \sin \phi_f , \quad (9)$$

the magnetic field, and

$$\bar{V} = v_p e^{i(\omega t - kz)} \hat{k} , \quad (10)$$

the acoustic velocity field, are at right angles.  $k$  represents the acoustic wave number. The source current will then be

$$\bar{J}_0 = \sigma \bar{V} \times \bar{F} , \quad (11)$$

or

$$J_{x0} = v_p F_y e^{i(\omega t - kz)} .$$

A Green's function method may be used to determine the effect that  $J_{x0}$  has on an arbitrary field point in spherical coordinates as follows. Define a vector potential  $\bar{A}$  such that  $\bar{B} = \Delta \times \bar{A}$ . Then

$$\bar{A}(\bar{r}) = \frac{\mu}{4\pi} \int \bar{J}_0(\bar{r}') \frac{e^{-\gamma|\bar{r} - \bar{r}'|}}{|\bar{r} - \bar{r}'|} dv' \quad (12)$$

where

$$\frac{e^{-\gamma|\bar{r} - \bar{r}'|}}{|\bar{r} - \bar{r}'|}$$



is the Green's function solution for the wave equation defined in equation (8),  $\bar{r}$  is the field position, and  $\bar{r}'$  is the source point. Taking  $\bar{r}$  to be zero (the origin of the coordinate system), and integrating over all space yields

$$A_x(0) = \frac{\mu}{\gamma^2 + k^2} \sigma v_p F_y e^{i\omega t}. \quad (13)$$

Incorporating the propagation of the wave along the z-axis into this solution yields the vector potential components

$$A_x(z) = \frac{\mu}{\gamma^2 + k^2} \sigma v_p F_y e^{i(\omega t - kz)} \quad (14)$$

and

$$A_y(z) = A_z(z) = 0.$$

Then

$$\bar{B} = \nabla \times \bar{A} = B_y \hat{j} \quad (15)$$

or

$$\begin{aligned} B_y &= \frac{i k \mu \sigma}{\gamma^2 + k^2} v_p F_y e^{i(\omega t - kz)} \\ &= \frac{\gamma^2}{\gamma^2 + k^2} \frac{v_p}{c} F_y e^{i(\omega t - kz)} \end{aligned} \quad (16)$$

where the acoustic wave number  $k$  has been taken as  $\omega/c$ , with  $c$  the velocity of sound in the medium. Also,

$$B_x = B_z = 0.$$

Eq. 16 gives a transverse propagating magnetic field induced by the propagating plane acoustic wave.

Two interesting factors appear in this equation: the first,  $\frac{v_p}{c}$ , is an acoustic mach number and introduces the magnitude of the acoustic wave into the output.

The second is the scaling factor,

$$\frac{\gamma^2}{\gamma^2 + k^2}.$$

In order to get a feeling for its effect, it is useful to introduce some approximations. First note the dependence on the conductivity and permittivity of the medium, and on the sound speed. For illustration, assume some typical values for these parameters from normal sea water and consider the frequency dependence of the scaling factor. If  $\sigma = 4$  siemens/meter,  $\mu = 4\pi \times 10^{-7}$  henries/meter, and  $c = 1500$  meters/second, then

$$\frac{\gamma^2}{\gamma^2 + k^2} = \frac{i\omega\mu\sigma}{i\omega\mu\sigma + \frac{\omega^2}{c^2}} = \frac{i}{(8.84 \times 10^{-2})\omega + i}. \quad (17)$$

The magnitude of Eq. 17 is plotted against frequency in Figure (1).

Note that at low frequencies, it is approximately unity, but decreases sharply from about 1 Hz to about 50 Hz, falling off approximately as  $1/\omega$ . This illustrates that the magnitude of the energy coupling will only be appreciable at low frequencies in a real ocean environment and will be very small above 50 Hz.

Consider the overall effect of an actual ocean environment. The earth's magnetic field can be assumed to have a value of approximately  $F = 5 \times 10^{-5}$  Tesla at mid-latitudes. For a plane acoustic wave with a pressure amplitude of 100 dB re 1  $\mu$ Pa,  $V_p = 6.7 \times 10^{-8}$  meters/second. Incorporating these values into equation (16) gives

$$B_y(f = 1 \text{ Hz}) = 1.94 \times 10^{-15} e^{i(\omega t - kz)} \text{ Tesla,}$$

and

$$B_y(f = 100 \text{ Hz}) = 3.99 \times 10^{-17} e^{i(\omega t - kz)} \text{ Tesla.}$$

The decrease in coupling effect with frequency is readily apparent. Also, note Bird's<sup>4</sup> pseudoradiation effect: the

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<sup>4</sup>Ibid.

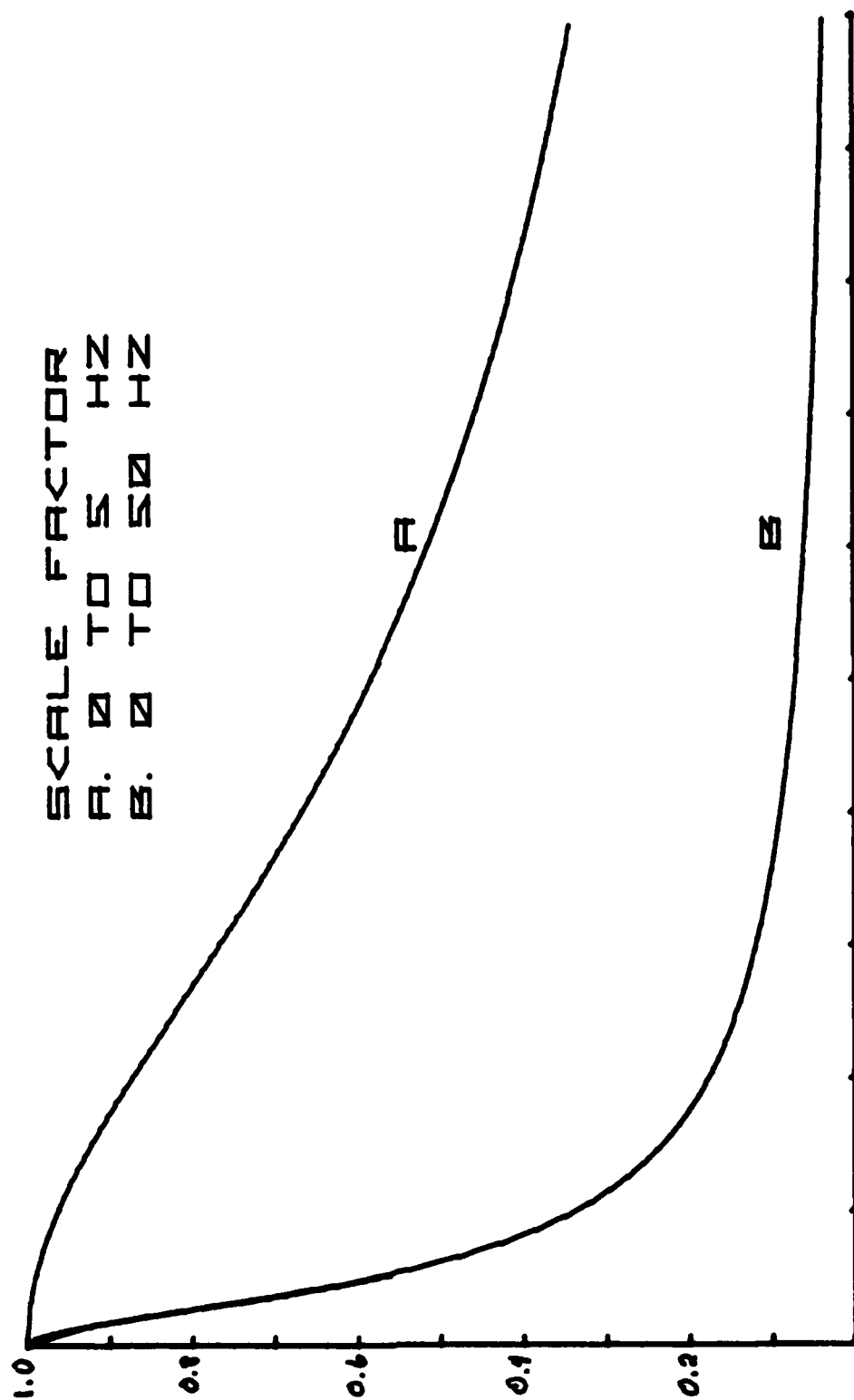


Figure 1. Scaling Factor

"propagation" of the magnetic wave is the continuous generation of it by the propagating acoustic wave.

### III. THE EXPERIMENT

#### A. EXPERIMENTAL EQUIPMENT

The primary constraint on a laboratory experiment is space. This shifts the operating frequency considerably above the optimal range of the magneto-acoustic coupling as discussed in the previous section, since the reduced dimensions cannot contain the ULF acoustic wavelengths. A resonant cavity was selected to contain the acoustic wave for two reasons. First, it allows one to artificially produce long wavelengths in the cavity instead of dealing with the free-space wavelength, as would be the case for a pulsed source. Second, use of a sinusoidally-varying acoustic source will induce a sinusoidally-varying output signal in the presence of a DC magnetic field. This facilitates signal processing for detection of the output by permitting long integration times of the stable AC signal.

Because of the difficulty of achieving rigid boundaries in an enclosure for a liquid, especially with the constraint of non-magnetic construction required for this experiment, pressure-release boundary conditions were selected. However, this further increases the operating frequency for the experiment, since the lowest mode that will fit into a pressure-release cavity is the (1,1,1) mode (one half wavelength in each dimension, x, y, and z), whereas a (0,1,0) mode would fit into an identical but rigid enclosure. This shift in frequency away from the region of efficient coupling requires

increasing the amplitude of the acoustic and magnetic fields and the conductivity of the solution in order to achieve measureable signal levels.

A Varian magnet, capable of stable magnetic fields on the order of 0.7 Telsa with a  $5\frac{1}{2}$  inch pole gap, was selected as the source for the magnetic field. The overall configuration of the magnet constrained the cross-section of the acoustic tank to be approximately 0.1 by 0.3 meters. Plexiglass was selected as material for construction of the tank because it closely approximates a pressure-release surface on the exterior walls of the tank, it is relatively easy to fabricate, and it is non-magnetic. To simplify analysis of the acoustic wave, the tank was constructed as a rectangular cavity 0.1 by 1.5 by 0.17 meters (inside dimensions).

The acoustic source for the experiment was a cylindrical transducer, 0.15 meters in diameter. This transducer was obtained on loan from Naval Ocean Systems Center, San Diego, and it is capable of quite high source levels in the frequency range of the experiment. However, the cylindrical source necessitated construction of a cylindrical source cavity, to be connected along its radius with one end of the rectangular test cavity, in order to couple the radiated sound energy effectively into the region of the magnetic field (see Figure 2). Although optimal mutual resonance of the two cavities was never achieved for the modal solution sought, an acceptable resonant wave pattern was obtained in the far end of the test cavity.

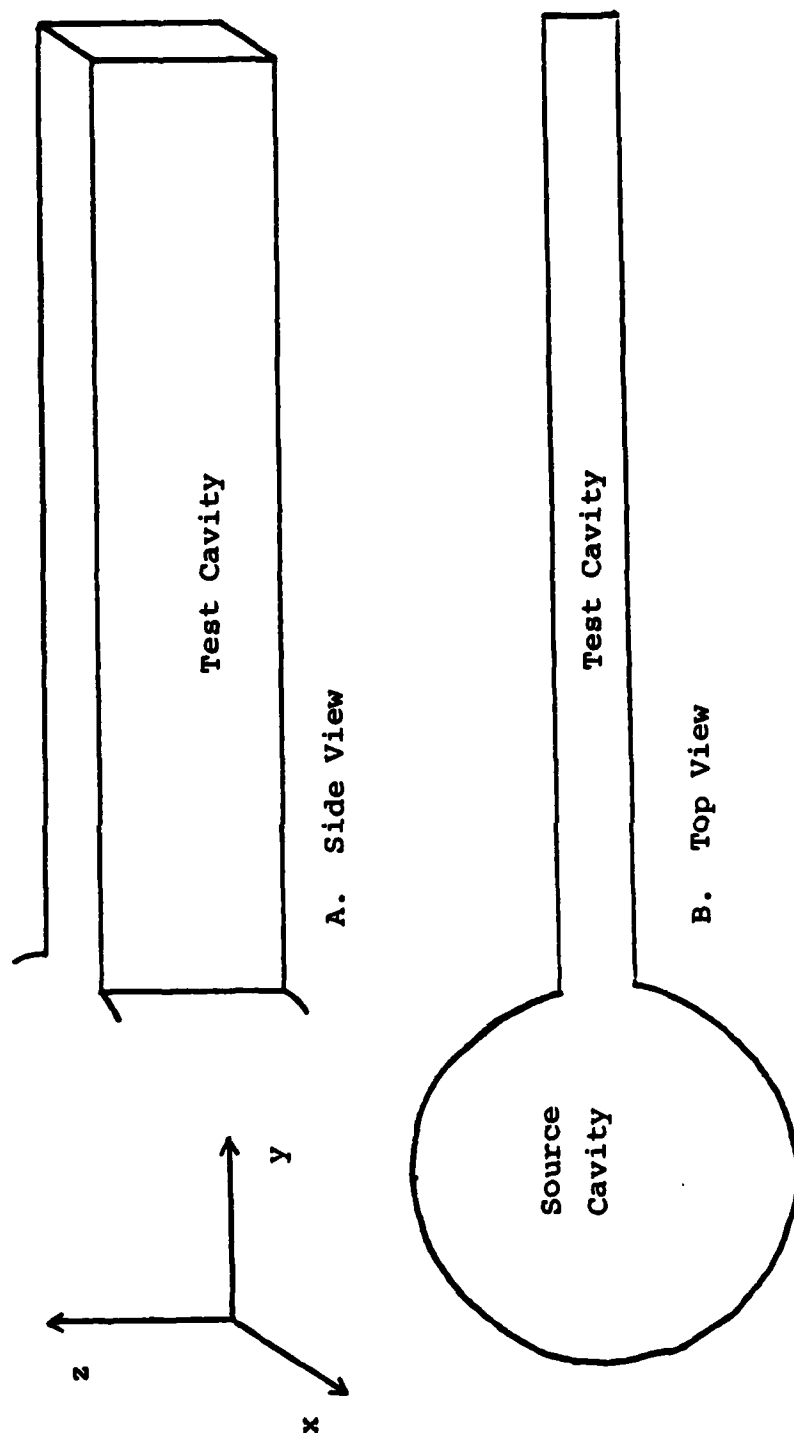


Figure 2. The Acoustic Wave Tank



A standing wave ratio of greater than ten-to-one was achieved with a half wavelength of about 0.305 meters. This acoustic field proved to be adequate for the experiment; see Appendix B for actual values achieved.

It was desired to obtain a low-order mode in the test tank; the first mode in the x and z dimensions (see Figure 3) for symmetry to facilitate analysis, and a half wavelength of greater than 0.3 meters in the y dimension in order to establish a uniformly polarized field within the influence of the magnet. The mode used for the experiment was a (1,5,1) mode (one half wavelength in the x and z dimensions and five half wavelengths in the y dimension) at about 10,350 Hz in salt water. This mode fulfilled the requirements of large amplitude, clean wave pattern, and acceptable wave dimensions for the experiment.

To further emphasize the interaction, a highly conductive medium was desired. Sea-salt saturated water was selected as the medium for its availability, but the conductivity achieved was so high that it was difficult to measure accurately with available equipment. An estimate was obtained using a conductivity bridge and a decade potentiometer, which, when corrected by comparison with the reading obtained for a standard potassium chloride solution, indicated a conductivity of 25 siemens/meter. The accuracy of this measurement was estimated at about  $\pm 20\%$ .

The experimental apparatus was set up as follows: A long rectangular plexiglass tank was filled with salt-saturated

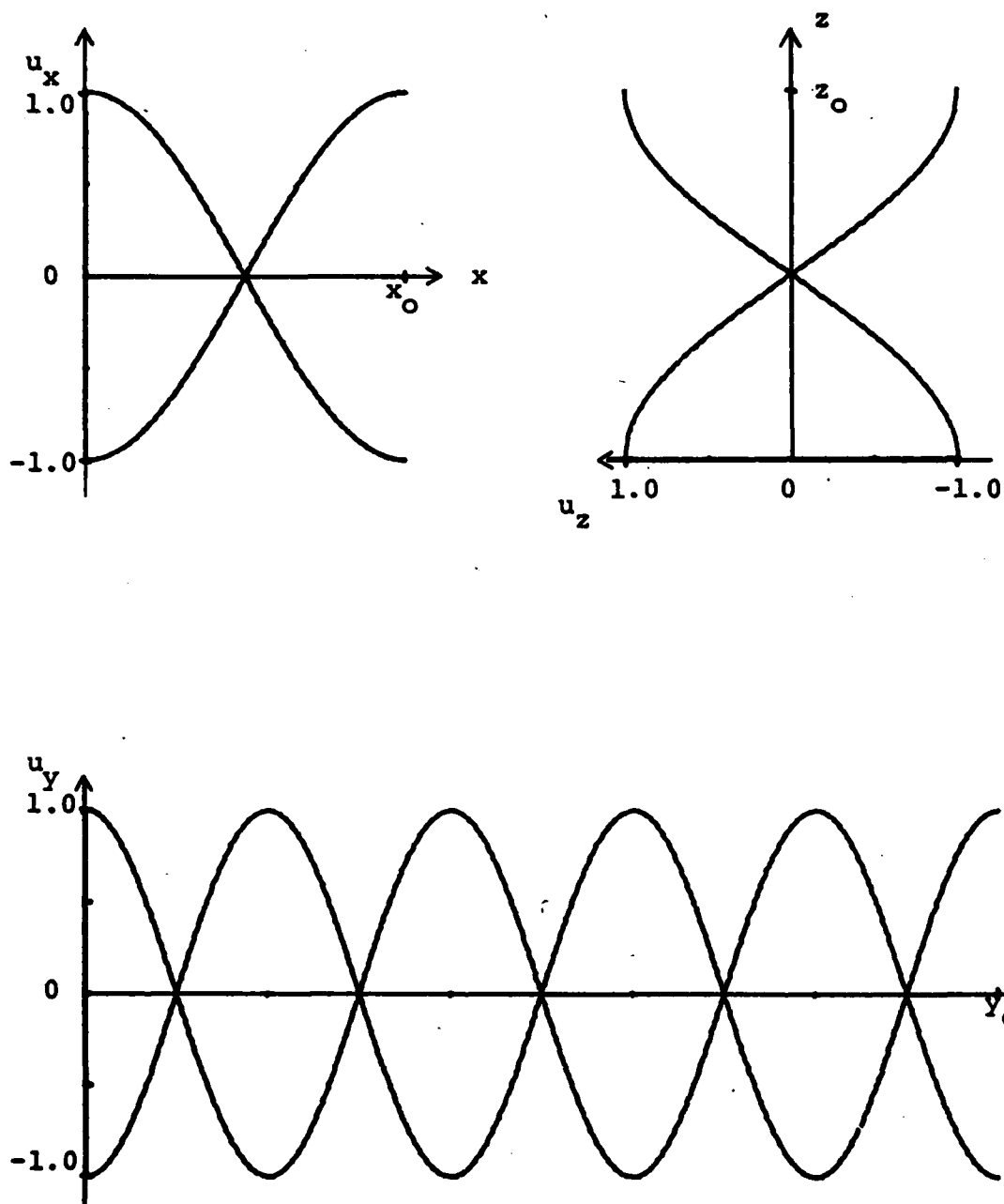


Figure 3. The Acoustic Standing Wave Patterns for the Experimental Mode; Normalized Velocity Amplitudes

water. An acoustic standing wave pattern was established in all three dimensions. A DC magnetic field was applied in the  $\pm x$  direction across the tank, centered on a pressure node (and therefore a velocity anti-node) along the y dimension of the tank. Two copper-clad rectangular plates (0.1 by 0.3 meters) were placed at the surface and the bottom of the tank, centered longitudinally on the axis of the magnet, with leads attached to an amplifier and spectrum analyzer to detect any electric fields or current that were induced (see Appendix A and Figures 4 and 5).

#### B. EXPERIMENTAL ANALYSIS

With the equipment established as described in the previous section, measurements were initiated to establish a noise level. A large coherent noise component at the frequency of the acoustic wave was collected at the receiving plates, even in the absence of a magnetic field. Observation of this noise spike indicated it was pickup from the power leads to the transducer. The receiver plates functioned as a relatively efficient antenna for the strong fields created by the acoustic source equipment. Various shielding and grounding efforts reduced, but could not eliminate, the noise spike. However, the noise proved to be stable, coherent, and reproducible.

Upon introduction of the DC magnetic field, the induced signal adds to the noise spike coherently. By comparing the magnitudes of the magnet-free noise spike with the magnitudes of the signal-plus-noise output for the magnet polarized in

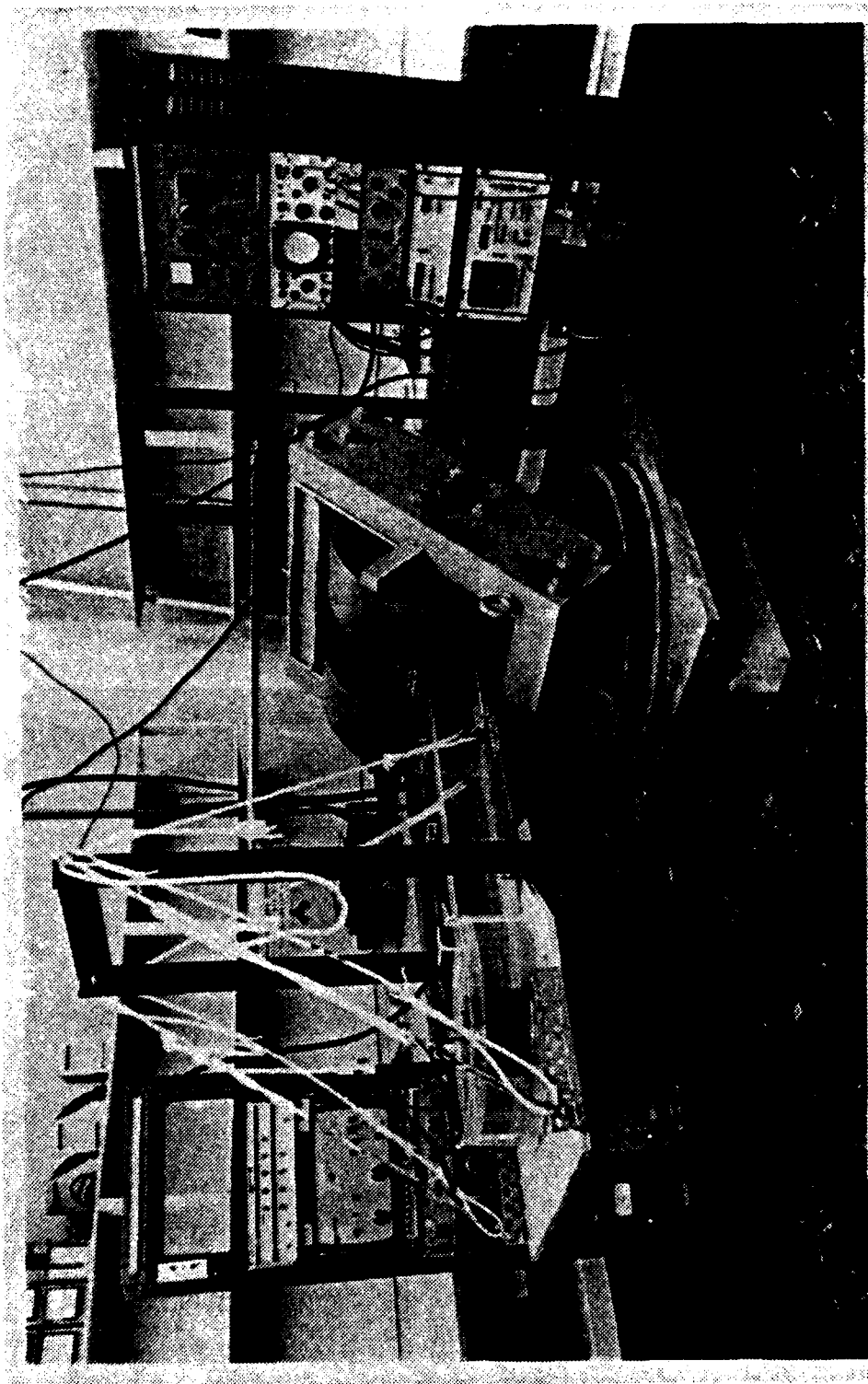


Figure 4. The Overall Experimental Apparatus

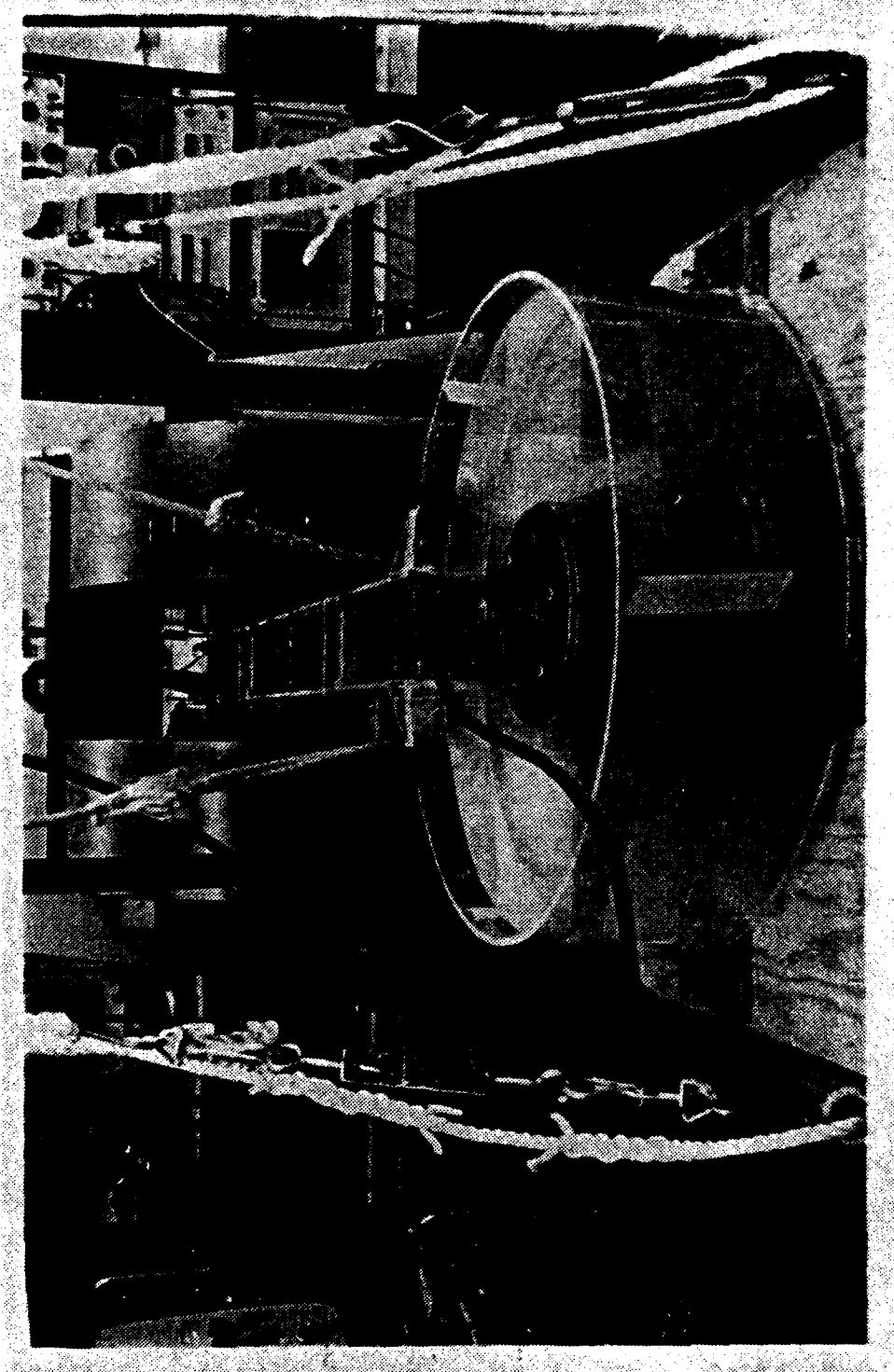


Figure 5. The Acoustic Tank Inside the Magnet

the "normal" and "reverse" modes, the magnitude of the induced signal is obtained as follows: By observing that the noise level is a consistent value, and that reversing the polarity of the magnet shifts the phase of the induced signal by  $180^\circ$  but does not affect its magnitude, two triangles with a common side can be constructed in a phasor diagram as shown in Figure 6.

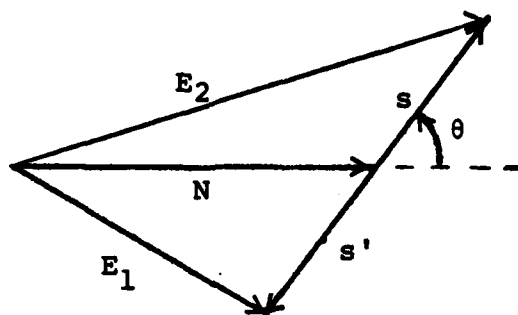


Figure 6. Phasor Resolution Diagram

The signal magnitude is then obtained by simultaneous solution of the law of cosines for each triangle:

$$\begin{aligned}
 E_2^2 &= N^2 + S^2 - (2NS) \cos(180 - \theta) \\
 E_1^2 &= N^2 + S^2 - (2NS) \cos(\theta)
 \end{aligned}
 \tag{18}$$

S and S' represent the signal phasors, N is the observed noise phasor (assumed here with zero phase angle), and  $E_1$

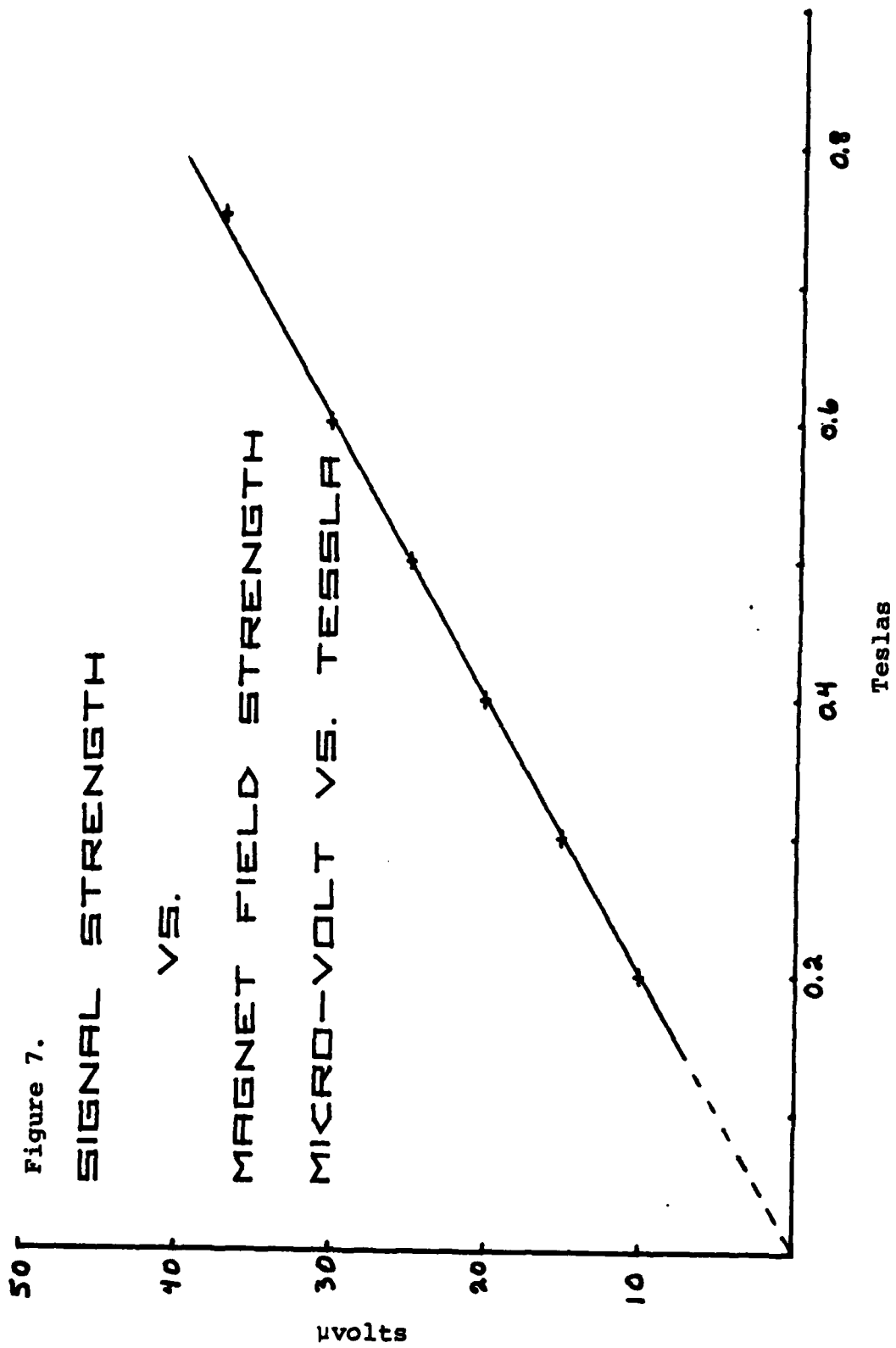
and  $E_2$  represent the observed voltages with the magnetic field applied in either polarity.

Once resolved, the induced signal was found to vary linearly with the strength of the magnetic field, at constant acoustic amplitude, as predicted. Initial runs were made using a General Radio Audio Oscillator, which showed a slight frequency drift. Although this did not affect the acoustic resonance, the spectrum analyzer integrating the drifting output introduced about 15% data scatter into the results. This problem was resolved by shifting to a crystal frequency synthesizer (see Appendix A) and integrating the output signal for approximately three minutes for each data point. Several of these runs were then averaged for each value used in the above equations; the results are plotted in Figure 7. A remarkably linear variation of signal output with magnetic field strength at a rate of 50  $\mu$ volt/Tesla was observed.

In order to determine if the observed signal is, indeed, the postulated magneto-acoustic coupling, it is necessary to calculate the magnitude of the acoustic particle velocity. The wave equation for the acoustic velocity potential is

$$\nabla^2 \phi - \frac{1}{c_0^2} \frac{\partial^2 \phi}{\partial t^2} = 0, \quad (19)$$

where  $c_0$  is the mean sound velocity and, by definition, the acoustic velocity and pressure are given by





$$\bar{u} = \nabla \phi \quad (20)$$

$$p = -\rho_0 \frac{\partial \phi}{\partial t}.$$

$\rho_0$  is the average fluid density. From standard separation of variables and applying pressure-release boundary conditions, a solution is obtained of the form

$$\phi = \phi_0 \sin(k_x x) \sin(k_y y) \sin(k_z z) \quad (21)$$

where

$$k_x = \frac{l\pi}{x_0}, \quad k_y = \frac{m\pi}{y_0}, \quad k_z = \frac{n\pi}{z_0} \quad (22)$$

are the associated wave numbers, and  $x_0$ ,  $y_0$ , and  $z_0$  are the effective cavity dimensions. Differentiation in the time domain introduces a factor of  $i\omega$ , so that

$$p = -\rho_0 \frac{\partial \phi}{\partial t} = -i\omega \rho_0 \phi \quad (23)$$

or

$$\phi = \frac{i p}{\omega \rho_0} \quad (24)$$

Using equations (19) above again yields

$$\begin{aligned}
\bar{u} &= \nabla \left[ \frac{i|p|}{\omega \rho_0} \sin(k_x x) \sin(k_y y) \sin(k_z z) e^{i\omega t} \right] \\
&= \frac{i|p|}{\omega \rho_0} e^{i\omega t} [k_x \sin(k_y y) \sin(k_z z) \cos(k_x x) \hat{i} \\
&\quad + k_y \sin(k_x x) \sin(k_z z) \cos(k_y y) \hat{j} \\
&\quad + k_z \sin(k_x x) \sin(k_y y) \cos(k_z z) \hat{k}]
\end{aligned} \tag{25}$$

The acoustic velocity field is of interest only insofar as it interacts with the magnetic field to produce a discernible signal, according to

$$\bar{E}_0 = \bar{V} \times \bar{B} . \tag{26}$$

Since the magnetic field is uniquely polarized in the  $\pm x$  direction, the cross-product with the x-component of the velocity field vanishes. Also, the z component will be a half wave cosine function at any point (in time and in the long dimension of the tank), and will therefore produce electrical components which are self-cancelling external to the tank. Therefore, the x and z components of the velocity field can be ignored for these calculations, and the relevant velocity field is reduced to

$$u_y = \frac{i|p|}{\omega \rho_0} k_y \sin(k_x x) \sin(k_z z) \cos(k_y y) e^{i\omega t} \tag{27}$$

Incorporating the known dimensions and modal solutions for the experiment, and averaging the result over the effective x, y, and z dimensions for the effective strength yields

$$\begin{aligned}\langle u_y \rangle &= 2.70 \frac{i|p|}{\omega \rho_0} \\ &= 5.39 \times 10^{-5} \text{ meters/second}\end{aligned}\tag{28}$$

from the observed parameters for the experiment, given in Appendix B.

To estimate the induced electric field strength of the magneto-acoustic interaction, couple this effective velocity magnitude with the DC magnetic field according to

$$\bar{E}_O = \bar{V} \times \bar{B} = u_y B \hat{k}\tag{29}$$

giving,

$$E_O = 53.9 \text{ } \mu\text{volt/meter/Tesla}.$$

This is equivalent, for the dimensions of the experiment, to an induced net voltage of  $V_O = 7.50 \text{ } \mu\text{volt/Tesla}$ , or about a factor of seven smaller than the observed signal voltage.

Another method of observing the magneto-acoustic coupling is to calculate the strength of the interaction as a current source. This approach attempts to compensate for leakage currents through the medium external to the source region.

Experimentally, this is accomplished by inserting known resistances  $R_s$  (shunt) in parallel with the receiver plates. From changes in output signal  $V_o$  with resistance, it is possible to calculate the apparent source strength  $I_o$  and leakage resistance  $R_l$  by simultaneous solution from

$$I_o = \frac{V_o}{R_l} + \frac{V_o}{R_s} \quad (30)$$

applied to several trials. This was carried out and the mean value of the induced apparent current source was approximately 400  $\mu$ Amp/Tesla. For comparison, equation (25) can be rewritten

$$\bar{J}_o = \sigma \bar{V} \times \bar{B} \quad (31)$$

or

$$\begin{aligned} I_o &= (\sigma \langle u_y \rangle B) \text{ (plate area)} \\ &= 41.1 \mu\text{Amp/Tesla,} \end{aligned} \quad (32)$$

which is again about an order of magnitude smaller than the observed values.

Several experimental variations were attempted in an effort to explore the nature of the interaction and to verify that the observed signal was in fact the result of the phenomenon of magneto-acoustic coupling. The acoustic source level was

varied by set amounts, and the observed signal levels, with constant magnetic field, varied linearly. Also, the experiment was repeated in fresh water with several combinations of magnetic field phase and magnitude, with no discernible signal from the plates (this eliminates plate motion as a possible source of the excess observed signal level).

The experiment has established that an electric field is induced by the presence of an acoustic field in a conductive medium with a DC magnetic field for the geometry of this experiment. Despite the discrepancy between the magnitude of the signal observed and the calculated expected magnitude, a magneto-acoustic interaction is the probable cause of this induced signal.

#### IV. CONCLUSIONS AND DISCUSSION

In the first section it was demonstrated that an acoustic wave propagating in a conductive medium in the presence of a strong magnetic field should induce electric potential fields and currents, and, associated with the electric currents, a magnetic field. It was noted that the magnetic field arising from the magneto-acoustic coupling should vary linearly with the strength of the acoustic wave and the magnetic field, is approximately inversely proportional to frequency, and is also dependent on the strength of the medium parameters, e.g., the sound velocity, permittivity, and conductivity. At ULF frequencies in an actual ocean environment, these phenomena are near the threshold of current detection capabilities and should therefore be discernible in an appropriately low noise field.

In the second section, an experiment was reviewed in which an electromagnetic signal was induced in a conductive salt solution by the interaction of a DC magnetic field with a resonant acoustic wave in a plexiglass cavity. The induced signal responded in the same manner as the expected signal from this coupling interaction, and its strength was within one order of magnitude of an approximate calculation. It seems reasonable to state on the basis of this evidence that the basic interaction has now been verified in the laboratory. ✓

There are a number of difficulties and questions left by this work, most evidently demonstrated in the disparity between experimental and theoretical magnitude.

Although the plexiglass cavity worked adequately for the experiment, as was remarked previously, it was still less than optimal. Significant difficulties were encountered in attempting to couple the two different geometry cavities together effectively. Most of the problem seems to be attributable to the impedance of the transducer face, an unknown in the design calculations. When mutual resonance of the two cavities was achieved (at higher frequencies and mode numbers that proved to be too short in wavelength for use in the experiment), an amplitude gain of 20 dB over the resonance mode used for the experiment was realized, along with higher standing wave ratios. The mode used for the experiment showed a good standing wave ratio and consistent sinusoidal wave patterns only at the end of the tank away from the source. The wave pattern in the region of coupling near the connection between the two cavities was muddled by evanescent modes. It is probable that a cleaner wave pattern would increase the sensitivity of the experiment considerably, and make the simplified Helmholtz solution used for the acoustic wave a more valid interpretation of experimental conditions.

Another area of compromise in the laboratory experiment was in the conductive fluid medium. Both the methods of mixing the salt solution and the methods of measuring the conductivity

of the result were limited to the materials and instruments on hand. The use of a pure salt (for example, sodium chloride or potassium chloride) would permit higher conductivities to be achieved and the conductivity of the result to be more consistent and predictable. Also, construction of a larger conductivity cell would enable more accurate measurements of a low-resistance solution than were possible with the instrumentation used for this experiment.

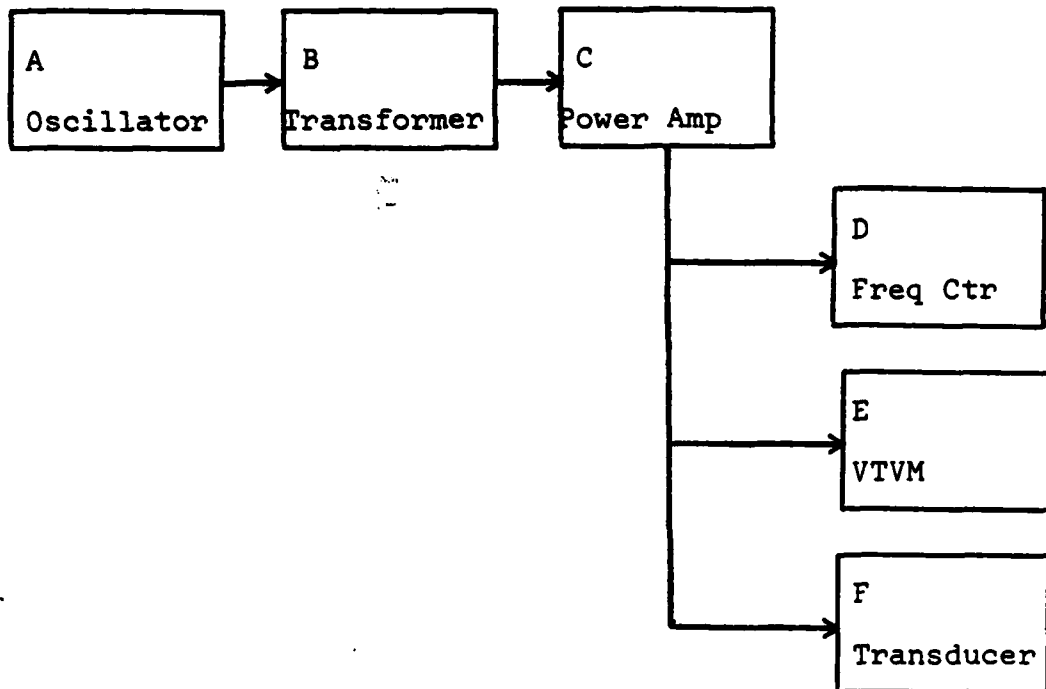
One other and more probable cause for the difference between the calculated and observed values of the induced currents is the method used to calculate the expected response. A more accurate potential function development for the fields occurring in and around the source region is necessary in order to precisely calculate the expected signal.

Another possible improvement on the experimental procedure would be to amplitude modulate the magnetic field sinusoidally, thereby inducing the magneto-acoustic signal at the sum and difference frequencies between the acoustic and magnetic frequencies. This would place the output away from the observed noise spike and remove any errors associated with the analysis of coherent addition between the signal and noise.

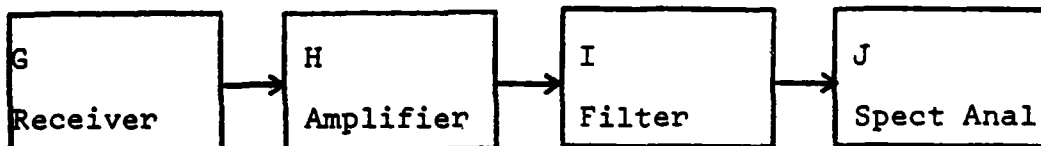
It is expected that further experimentation in this field will both confirm the results found here and expand the theory towards the goal of a practical application, as discussed in the Introduction.



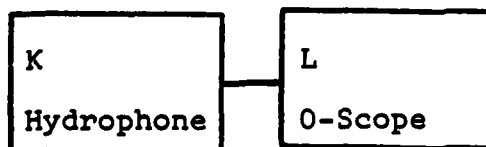
APPENDIX A  
Equipment Schematics



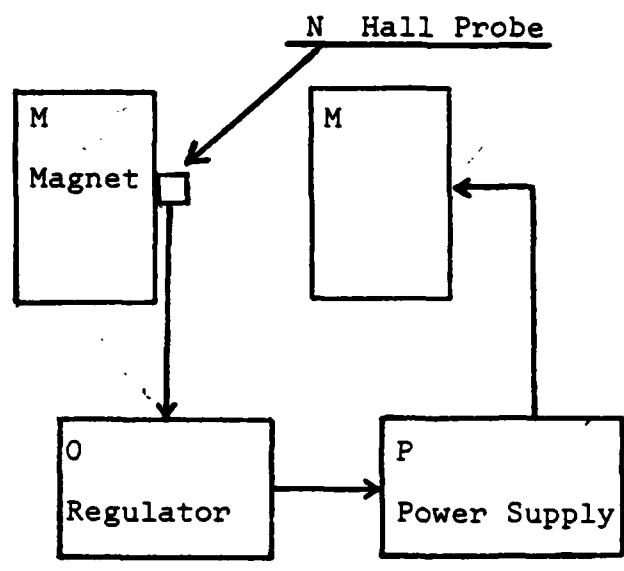
Acoustic Source



Signal Reception



Acoustic Reception



Magnet Control

## EQUIPMENT

- A. Audio Oscillator  
Hewlett Packard Frequency Synthesizer, Model 3320B
- B. Impedence Matching Transformer  
Krohn-Hite Matching Transformer, Model MT-56
- C. Power Amplifier  
Krohn-Hite 50 Watt Amplifier, Model DCA-50
- D. Frequency Counter  
Hewlett Packard Electronic Counter, Model 521C
- E. VTVM  
Hewlett Packard Vacuum Tube Voltmeter, Model 400D
- F. Transducer  
(NUSWC) B78CR8M
- G. Receiver  
Copper-clad Fiberglass plates, 0.1 X 0.3 m
- H. Voltage Amplifier  
Hewlett Packard Amplifier, Model 465A  
(set at +40 dB throughout experiment)
- I. Filter  
Krohn-Hite Filter, Model 3750  
(settings: High-pass, 8,200 Hz cut-off, 18 dB/octave)
- J. Specrum Analyzer  
EMR Schlumberger Digital Spectrum Analyzer, Model 1510  
EMR Schlumberger Digital Spectrum Translator, Model 1520
- K. Hydrophone  
Model LC-10 Transducer
- L. Oscilloscope  
Tektronix Oscilloscope, Type RM 503
- M. Magnet  
Varian Associates Electromagnet, Model V-4012-3B
- N. Hall Effect Probe
- O. Regulator  
Fielddial Magnetic Field Regulator, Model V-FR-2100
- P. Magnet Power Supply  
Varian Associates Regulated Magnet Power Supply, Model V 2100 B

## APPENDIX B

### EXPERIMENTAL PARAMETERS

Test tank dimensions, in meters,

$$x_o = 0.1 \text{ meters}$$

$$y_o = 1.502 \text{ meters}$$

$$z_o = 0.139 \text{ meters}$$

Acoustic wave numbers ( $k_n$ ) for the (1,5,1) mode at 10,350 Hz,  
in inverse meters,

$$k = 43.35$$

$$k_x = 31.42$$

$$k_y = 10.46$$

$$k_z = 22.60$$

Acoustic pressure amplitude, as measured with an LC-10  
hydrophone with a sensitivity of -108.8 dB re 1 volt per  
microbar, in Pascals,

$$p = 2.15 \times 10^3$$

$$p_{\text{RMS}} = 1.52 \times 10^3$$

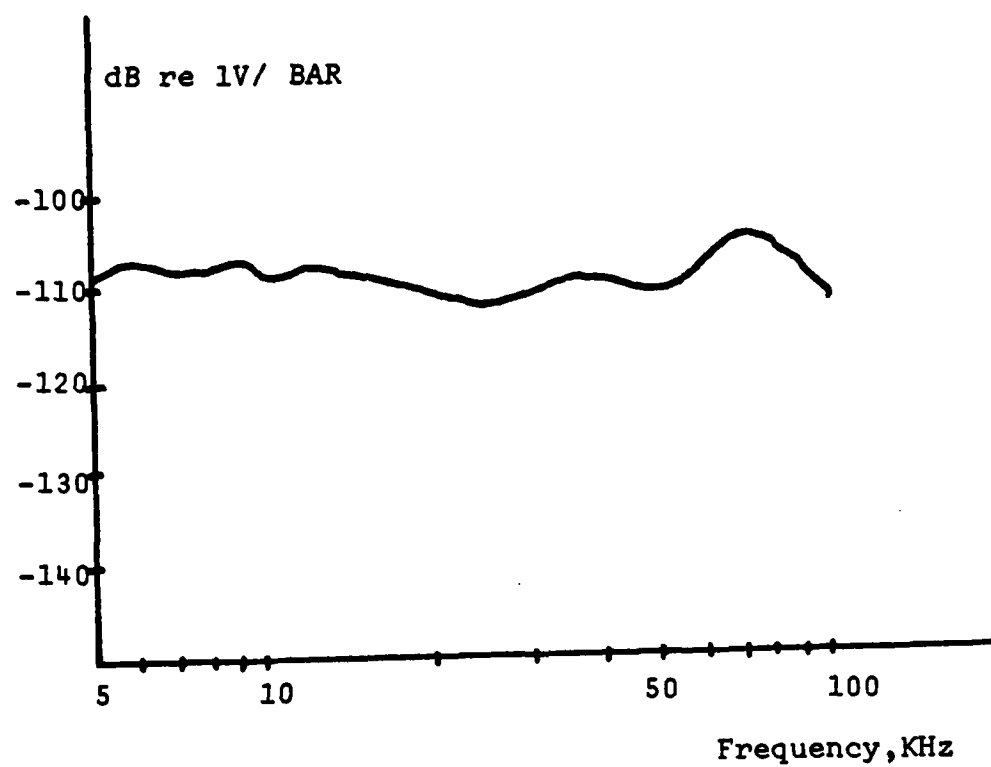
Fluid Density

$$\rho_o = 1.17 \times 10^3 \text{ kg/m}^3$$

Fluid conductivity

$$\sigma = 25.4 \text{ siemens/meter}$$

APPENDIX C  
Calibration Data



Sensitivity of the LC-10 Transducer, Serial No. 1922



Transmitting Response of the B78CR8M Transducer

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3. Bird, J. F., "Hydromagnetism Induced by Submerged Acoustic Sources: Sonomagnetic Pseudo Radiation." JASA. November, 1977.

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